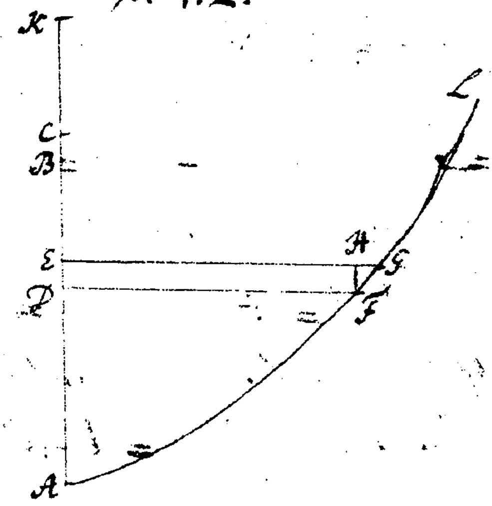
A. 42.



Ad axem verticalem CDA invenire & construere curvam AFG ejus naturae, ut mobile decidens per axeos portionem quamcunque BA impetu concepto ascendat per arcum AF aequalem altitudini unde deciderat AB, idque in medio resistente secundum quadrata velocitatum.

Incipiat mobile descendere in loco axeos K; sit $AB = x$, $BC = -\partial x$, velocitas in puncto quocunque C, $= v$, vis gravitatis ordinaria $= g$, tempusculum quo percurritur CB, $= \partial t$. Ut habetur vis acceleratrix per elementum CB, $= g - v^2$, quam multiplicando in particulam temporis ∂t habetur incrementum velocitatis $\partial v = (g - v^2)\partial t$, adeoque $\partial t = \frac{\partial v}{g - v^2}$ sed $\partial t = \frac{-\partial x}{v}$, quare $-\frac{\partial x}{v} = \frac{\partial v}{g - v^2}$, & $-2\partial x = \frac{2v\partial v}{g - v^2}$, sumptis integralibus, $2x = l\,\overline{g - v^2}$. Jam in puncto K ubi $x = AK = a$, est $v = 0$, ~~quare~~ posita autem in aequatione inventa $x = a$ & $v = 0$ habetur $2a = lg$, quare ut aequatio debite corrigatur, addatur ad primum aequationis membrum lg & ad secundum $2a$; unde $2x - 2a = -lg + l\,\overline{g - v^2}$. Fiat jam $x = 0$, fietque in vertice seu puncto infimo A, $-2a = -lg + l\,\overline{g - v^2}$, seu $2a = l\frac{g}{g - v^2}$; assumtaque c pro numero unitatis $c^{2a} = \frac{g}{g - v^2}$; $c^{2a}g - c^{2a}v^2 = g$; $v^2 = \frac{c^{2a}g - g}{c^{2a}}$, tandemque $v = \sqrt{\frac{c^{2a}g - g}{c^{2a}}}$; sumtisque differentiis posita a variabili, $\partial v = \frac{\partial a\sqrt{g}}{c^{2a}\sqrt{c^{2a} - 1}}$

Sit curvae AFL abscissa $AD = y$, arcus $AF = z$, $DE = FH = \partial y$, $FG = \partial z$ vis retardatrix a gravitate oriunda est $\frac{HF}{FG} \cdot g = \frac{\partial y}{\partial z} g$, cui addendo vim resistentiam v^2, erit vis tota retardatrix in puncto $F = \frac{\partial y}{\partial z} g + v^2$, quam multiplicando per elementum temporis $\frac{\partial z}{v}$, habetur decrementum velocitati $-\partial v = (\frac{\partial y}{\partial z} g + v^2) \times \frac{\partial z}{v}$, unde $v^2 \partial z + g\partial y = -v\partial v$, vel pro ∂z scribendo $p\partial y$, $v^2 p\partial y + g\partial y = -v\partial v$. Pro integranda hac aequatione, fiat per methodum Bernoullianam $\frac{1}{2}v^2 = mn$, unde $-v\partial v = -m\partial n - n\partial m$. Sit $v^2 p\partial y = -n\partial m = -\frac{\frac{1}{2}v^2 \partial m}{m}$, & dividendo per v^2

$p\partial y = -\frac{1}{2}\frac{\partial m}{m}$; $\int p\partial y = -\frac{1}{2} l m = l\frac{1}{\sqrt{m}}$ & $c^{\int p\partial y} = \frac{1}{\sqrt{m}}$, vel $m = \frac{1}{c^{2\int p\partial y}}$

In aequatione altera $g\partial y = -m\partial n$, substituatur hic valor ipsius m eritque $g\partial y = \frac{-\partial n}{c^{2\int p\partial y}}$, vel $-\partial n = c^{2\int p\partial y} g\partial y$, adeoque $n = gb - \int c^{2\int p\partial y} g\partial y$ tandemque $mn = \frac{1}{2}v^2 = \frac{1}{c^{2\int p\partial y}}\left(bg - \int c^{2\int p\partial y} g\partial y\right)$. Jam in vertice curvae A evanescit $\int p\partial y = z$, & $\int c^{2\int p\partial y} g\partial y$, quare in hoc casu est $\frac{1}{2}v^2 = bg$, adeoque $v = \sqrt{2bg}$ Sed per demonstrata est in vertice curvae $v = \sqrt{\frac{c^{2a} - 1 \cdot g}{c^{2a}}}$ quare $\sqrt{2bg} = \sqrt{\frac{c^{2a} g - g}{c^{2a}}}$, vel $2b = \frac{c^{2a} - 1}{2c^{2a}}$, substituto itaque hoc valore ipsius b, $\frac{1}{2}v^2 = \frac{1}{c^{2\int p\partial y}}\left(\frac{c^{2a} - 1}{2c^{2a}} g - \int c^{2\int p\partial y} g\partial y\right)$. Ut innotescat punctum quietis L, seu punctum ultimum quo mobile in curva ascendit fiat $v = 0$, & habebitur $\frac{c^{2a} - 1}{2c^{2a}} = \int c^{2\int p\partial y} \partial y$, sumtisque differentiis

$\frac{\partial a}{2c^{2a}} = c^{2z}\partial y. = \frac{c^{2z}}{p}\partial z$. Sed ob conditionem problematis $a = z$ et $\partial a =$ unde $\frac{1}{2c^{2z}} = \frac{c^{2z}}{p}$, et $p = c^{4z} = \frac{dz}{dy}$, hinc $4z = l\frac{dz}{dy}$; idquod aequi

unde $\frac{1}{2c^{2a}} = \frac{c^{2z}}{p}$. $p = 2c^{a+2z} = \frac{\partial z}{\partial y}$, $lz + a + 2z = l\frac{\partial z}{\partial y}$ qualitas arcus curvae quaesitae proportionalis esse logarithmo rationis $dz\,ady$, vel

Est vero $c^{2a} - 1 = 2c^{2a}\int c^{2z}\partial y$, adeoque $c^{2a} = \frac{1}{1 - 2\int c^{2z}\partial y}$, & hoc valore substituto $1 - 2\int c^{2z}\partial y = \frac{c^{2z}}{p} = (\text{ob } p = \frac{\partial z}{\partial y}) \frac{c^{2z}\partial y}{\partial z}$, adeoque $\partial z = \frac{c^{2z}\partial y}{1 - 2\int c^{2z}\partial y}$ unde $z = \frac{1}{2} l(1 - 2\int c^{2z}\partial y)$, tandemque $c^z = \sqrt{1 - 2\int c^{2z}\partial y}$

Ut separentur indeterminatae, sumantur differentiae $2c^{2z}\partial z = -2c^{2z}\partial y$, & div. per $-2c^{2z}$, $\partial z = + c^{4z}\partial y$, $+\partial y = \frac{+\partial z}{c^{4z}}$. Sumantur iterum integralia $e + y = \frac{-1}{4c^{4z}}$, unde $c^{4z} = \frac{-1}{4e + 4y}$, sumtisque logarithmis $4z = l\frac{-1}{4e + 4y}$, $4\partial z = \frac{-\partial y}{e + y}$

$$\partial z^2 = \partial y^2 + \overset{dt^2}{\cancel{\partial x^2}} = \frac{\partial y^2}{16(e+y)^2}\,.\ \overset{dt^2}{\cancel{\partial x^2}} = \partial y^2\left(\frac{1}{16(e+y)^2} \mp 1\right) = \partial t^2.$$

$$\cancel{\partial t = \partial y\sqrt{\frac{1-y^2}{y^2}} = y^{-1}\partial y\sqrt{1-y^2}} = \frac{\partial y}{y}\sqrt{b^2-y^2}$$

$$dt = dy\sqrt{\left(\frac{1}{16(e+y)^2} - 1\right)} = dy\sqrt{\frac{1-16ee-32ey-16yy}{16(e+y)^2}} = \frac{dy}{4e+4y}\sqrt{1-16ee-32ey-16yy}$$

vel posita arbitraria $e=0$, $dt = \frac{dy}{4y}\sqrt{1-16yy} =$ (supplendo homogeneam) $\frac{dy}{4y}\sqrt{16bb-16yy}$ $= \frac{dy}{y}\sqrt{bb-yy}$.

Catenariae immediate ad aequationem quaesitam pervenitur differentiando simplicitur $4z = l\,\frac{dz}{dy}$; posita enim dy constante, habetur $4dz = \frac{ddz}{dz}$, vel, multiplicando per $\frac{dy}{dz}$, $4dy = \frac{dy\,ddz}{dz^2}$; integrando erit $4y = \cancel{4b}\ \frac{-dy}{dz} =$ (ad observandam homogeneitatem) $\frac{4b\,dy}{dz}$, hinc $dz = \frac{b\,dy}{y}$ et $dz^2 = (\cancel{4}\,dy^2 + dt^2 =)\ \frac{bb\,dy^2}{yy}$; ergo $dt = \frac{dy}{y}\sqrt{bb-yy}$.

ergo $c^{-2z} = 1 - 2\int c^{2z}dy$